

Coefficients de Clebsch-Gordan - usando uma tabela

Exemplos:

① Coluna sombreada na tabela 2×1 ($j_1=2, j_2=1$)

$$|3,0\rangle = \sqrt{\frac{1}{5}} |2,1\rangle |1,-1\rangle + \sqrt{\frac{3}{5}} |2,0\rangle |1,0\rangle + \sqrt{\frac{1}{5}} |2,-1\rangle |1,1\rangle$$

$\begin{matrix} \uparrow & \uparrow & \uparrow & \uparrow \\ j_1 m_1 & j_2 m_2 & & \end{matrix}$

TABLE 4.8: Clebsch-Gordan coefficients. (A square root sign is understood for every entry; the minus sign, if present, goes *outside* the radical.)

$$1/2 \times 1/2$$

1
+1/2 +1/2
-1/2 +1/2
-1/2 -1/2

$$1 \times 1/2$$

3/2
+3/2
+1 +1/2
+1 -1/2
0 +1/2
0 -1/2
-1 +1/2
-1 -1/2

$$2 \times 1$$

3
+3
+2 +1
+2 0
+1 +1
+2 -1
+1 0
0 +1

$$1 \times 1$$

2
+2
+1 +1
+1 0
0 +1
+1 -1
0 0
-1 +1

1/6 1/2 1/3
2/3 0 -1/3
1/6 -1/2 1/3
0 -1
-1 0
0 -1
-1 -1

j_1
 j_2

$$3/2 \times 1$$

5/2
+5/2
+3/2 +1
+3/2 0
+1/2 +1

3
0
0
1/5 1/2 3/10
3/5 0 -2/5
1/5 -1/2 3/10
0 -1
-1 0
-2 +1

$$2 \times 1/2$$

5/2
+5/2
+2 1/2
+2 -1/2
+1 +1/2
0 +1/2
0 -1/2
-1 +1/2
-2 +1/2
-2 -1/2

$$3/2 \times 1/2$$

2
+2
+3/2 +1/2
+3/2 -1/2
+1/2 +1/2
1/4 3/4
3/4 -1/4
2 1
0 0
1/2 1/2
1/2 -1/2
2 1
-1 -1
-1/2 -1/2
-3/2 +1/2
3/4 1/4
1/4 -3/4
2 -2
-3/2 -1/2
1

5/2 3/2
2/5 3/5
3/5 -2/5
1/10 2/5 1/2
3/5 1/15 -1/3
3/10 -8/15 1/6
5/2 3/2
-1/2 -1/2
3/10 8/15 1/6
3/5 -1/15 -1/3
1/10 -2/5 1/2
5/2 3/2
-3/2 -3/2

3/10 8/15 1/6
3/5 -1/15 -1/3
1/10 -2/5 1/2
-1/2 -1
-3/2 0
3/5 2/5
2/5 -3/5
5/2
-5/2
-3/2 -1
1

2/3 1/3
1/3 -2/3
3 -3
-2 -1
1

② Linha sombreada da tabela $3/2 \times 1$ ($j_1=3/2, j_2=1$)

$$|3/2, 1/2\rangle |1,0\rangle = \sqrt{\frac{3}{5}} |5/2, 1/2\rangle + \sqrt{\frac{1}{15}} |3/2, 1/2\rangle - \sqrt{\frac{1}{3}} |1/2, 1/2\rangle$$

$\begin{matrix} \uparrow & \uparrow & \uparrow & \uparrow \\ j_1 m_1 & j_2 m_2 & j & m \end{matrix}$